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A new dimensionless group model for determining the viscosity of nanofluids

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Abstract This article presents a model, based on dimensionless groups, to predict the viscosity of nanoparticle suspensions, nanofluids. This empirical model expresses the viscosity of a nanofluid as a function of the following: viscosity of the base liquid, particle volume fraction, particle size, properties of the surfactant layer, and temperature. According to this model, viscosity changes nonlinearly with nanoparticle loading. Compared to other models, the new model is in good agreement with experimentally determined viscosity data for alumina–water nanofluids.

Keywords Dimensionless group · Nanofluid · Viscosity · Nanoparticle suspension

Introduction

Heat-transfer fluids (e.g., water, ethylene glycol, engine oil, etc.) have an important role in society. They are routinely used in chemical production, manufacturing, power generation, transportation, and many other aspects of modern life. Through enhancing the thermal properties of heat-transfer fluid, more efficient systems and devices could be built. The development of high-performance, heat-transfer suspensions has been the subject of numerous studies over the last few decades [1, 2]. Suspensions containing particles from micrometers to millimeters in size have shown improved thermal properties. Unfortunately, these suspensions are also unstable and prone to clogging systems with small channels [3–6]. Recent advances in nanotechnology have permitted nanofluids to be developed. Nanofluids are suspensions containing nanoparticles, particles having diameters less than 100 nm. Nanofluids have proven to have high-thermal conductivities. Unlike suspensions containing larger particles, nanofluids have proven to be stable and do not clog even micron-sized channels. Others have demonstrated that nanofluids, containing small amounts of metal or nonmetal nanoparticles, such as Cu [7], Ag [8], Au [8], CuO [9], Al_2O_3 [10, 11], TiO_2 [12], or carbon nanotubes [13] exhibit substantially enhanced thermal conductivities compared to that of the base fluid.

Most studies in the field of nanofluids have focused on characterizing the thermal conductivities of the fluid. For heat-transfer applications, high-thermal conductivity is not the only factor that must be considered. The viscosity of the fluid is also an important factor. The viscosity of the fluid affects resistance to flow and convective heat transfer. As the viscosity of a nanofluid increases as particle concentration increases [14], low-particle concentrations are envisaged for use in heat-transfer applications. Only a few works have been found that discuss the viscosity of nanofluids [15–18]. The complete lack of experimental data in the literature is even more striking.

Various models have been developed to predict the viscosity of a particle suspension. One of the earliest of these models was developed by Einstein in 1906:

$$\eta_{\mathrm{nf}} = \eta_{\mathrm{bf}}(1 + K_{\mathrm{H}}\phi), \tag{1}$$

where η_{nf} and η_{bf} are the viscosities of the suspensions and base fluid, K_{H} the shape factor of the particle, and ϕ the

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volume fraction of particles in the suspension. This model neglects any effect that the particle size may have, but does consider particle shape and concentration effects. For spherical particles, shape factor of 2.5 is used [19, 20]. Equation 1 is valid if there are negligible particle–particle interactions. At higher particle concentrations, these interactions become important, and the hydrodynamic volume fraction, ϕ_h, is used:

$$\phi_h = \phi\left[\frac{d+2s}{d}\right]^3, \tag{2}$$

where d is the particle diameter, and s the thickness of the capping layer [20]. Brinkman [21] extended the Einstein model to accommodate moderate particle volume concentrations. For particle volume fractions less than 4%, the Brinkman model is as follows:

Table 1 Empirical constants for Al_2O_3–water systems determined by least-squares regression

m	α	β	γ
0.72	−0.485	14.94	0.0105

$$\frac{\eta_{nf}}{\eta_{bf}} = \frac{1}{(1-\phi)^{2.5}}. \tag{3}$$

The Batchelor model [22] considers the effects of Brownian motion for spherical particles:

$$\frac{\eta_{nf}}{\eta_{bf}} = \left(1 + 2.5\phi + 6.5\phi^2\right). \tag{4}$$

Empirical viscosity models have been developed for suspensions containing nanoparticles of a particular size. Nguyen et al. [23] developed the following viscosity correlations for 36 and 47 nm particles, respectively:

$$\frac{\eta_{nf}}{\eta_{bf}} = \left(1 + 0.025\phi + 0.015\phi^2\right), \tag{5}$$

$$\frac{\eta_{nf}}{\eta_{bf}} = 0.904e^{0.1483\phi}. \tag{6}$$

Both of these models determine the viscosity by only considering the viscosity of the base fluid and the particle volume fraction. In this work, a model is presented to estimate the viscosity of a nanofluid. The model is based on dimensionless groups and uses a least-squares regression technique to determine model parameters [24, 25].

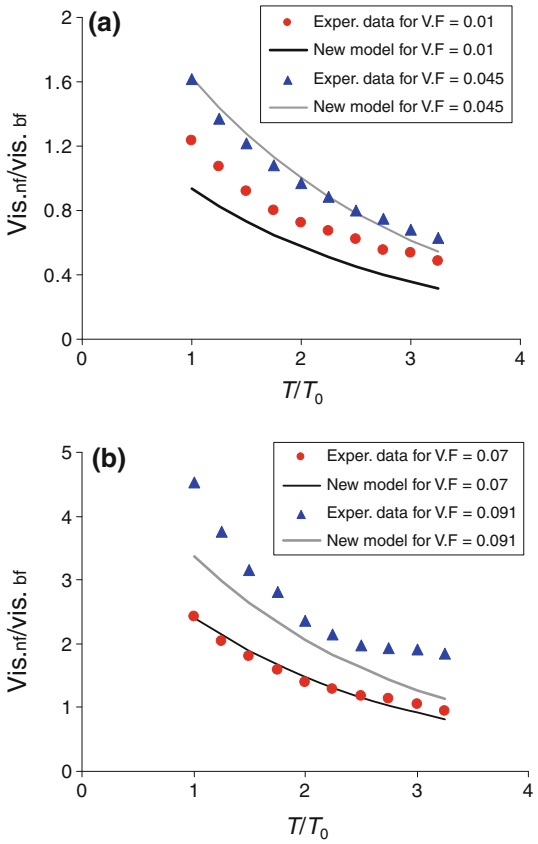

Fig. 1 Model prediction and experimental data for Al_2O_3–water nanofluid: **a** $d = 36$ nm, $\phi_h = 1$ and 4.5%, **b** $d = 36$ nm, $\phi_h = 7$ and 9.1%

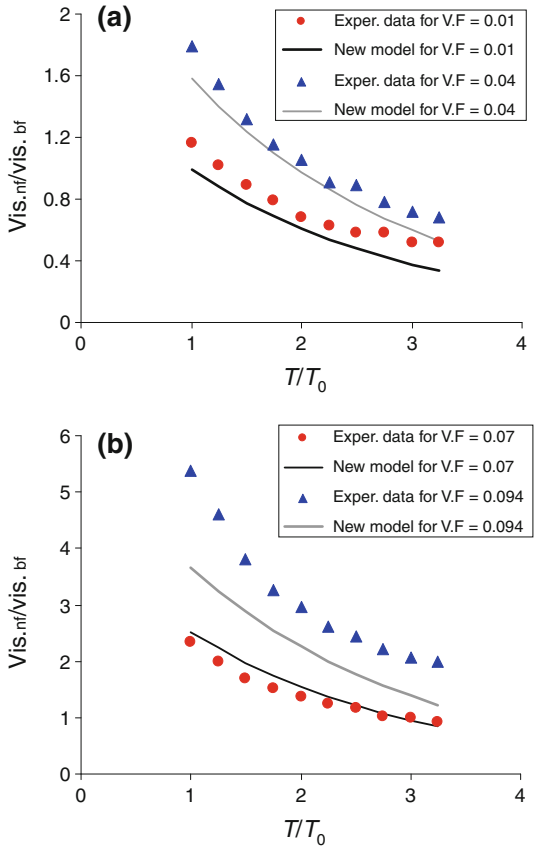

Fig. 2 Model prediction and experimental data for Al_2O_3–water nanofluid: **a** $d = 47$ nm, $\phi_h = 1$ and 4%, **b** $d = 47$ nm, $\phi_h = 7$ and 9.4%

The proposed new model for viscosity of nanofluids

Through the use of appropriate dimensionless groups, a new expression to estimate the viscosity of nanofluids will be derived in this section. The viscosity of the nanofluid, η_{nf}, will be considered to be a function of various dimensionless groups. These groups are chosen so that the viscosity of the base fluid, the hydrodynamic volume fraction of nanoparticles, diameter of the nanoparticle, thickness of capping layer on the nanoparticle, and changes in temperature are all taken into consideration. The following dimensionless groups are thus defined:

$$\pi_1 = \frac{\eta_{nf}}{\eta_{bf}}, \quad \pi_2 = \phi_h, \tag{7,8}$$

$$\pi_3 = \frac{d}{1+r}, \quad \pi_4 = \frac{T}{T_0}, \tag{9,10}$$

where η_{nf} is the viscosity of the nanofluid, η_{nb} the viscosity of the base fluid, ϕ_h the hydrodynamic volume fraction of solid nanoparticles, d the nanoparticle diameter, r the thickness of the capping layer, T_0 a reference temperature, and T the measured temperature of the nanofluid. One of the dimensionless groups is taken to be a function of the other groups as follows:

$$\frac{\eta_{nf}}{\eta_{bf}} = f_1\left(\phi_h, \frac{d}{1+r}, \frac{T}{T_0}\right). \tag{11}$$

The function, f_1, is expressed as follows:

$$\frac{\eta_{nf}}{\eta_{bf}} = \exp\left[m + \alpha\left(\frac{T}{T_0}\right) + \beta(\phi_h) + \gamma\left(\frac{d}{1+r}\right)\right], \tag{12}$$

where m is a factor that depends on the properties of the system (i.e., the solid nanoparticles, the base fluid, and their interactions), while α, β, and γ are empirical constants determined from experimental data. The empirical constants, as described in Eq. 12, were calculated from a set of experimental data for alumina–water nanofluids [23] using least-squares regression. The results from this analysis can be seen in Table 1. For this system, a reference temperature of $T_0 = 20$ °C and a capping layer thickness of $r = 1$ nm were used.

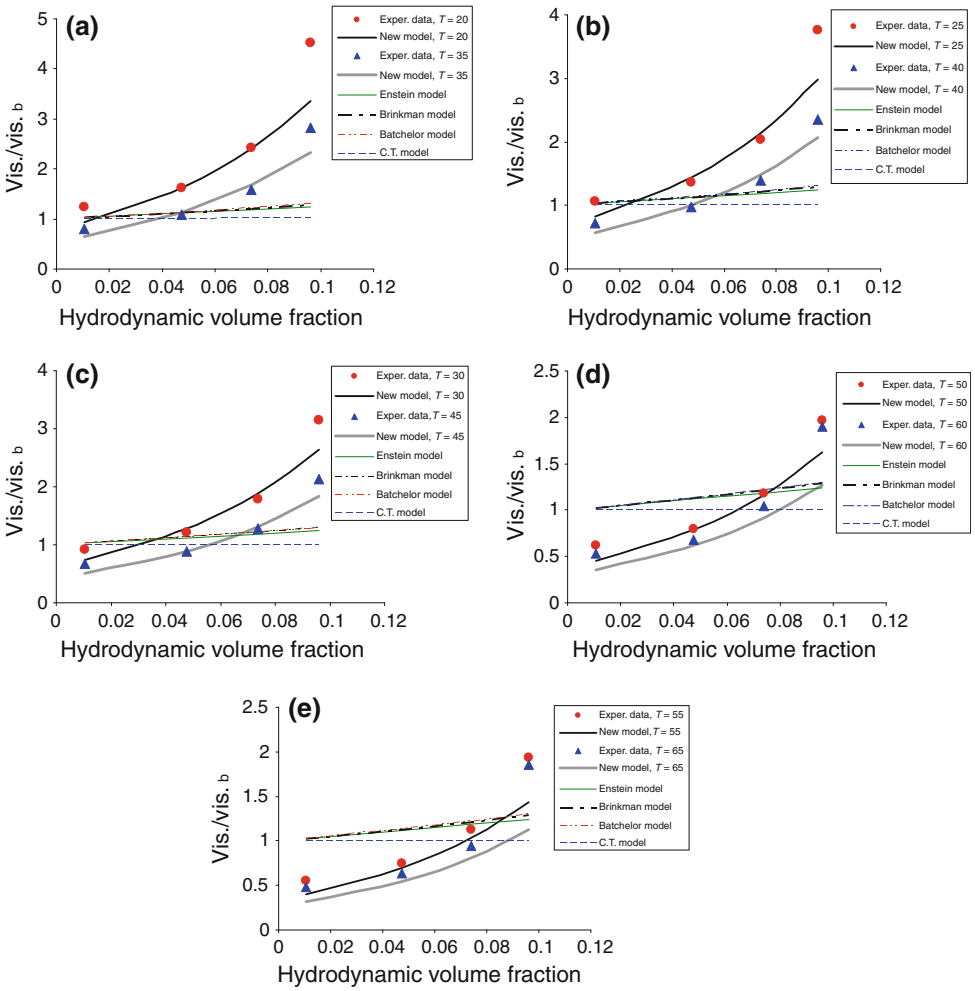

Fig. 3 Model prediction and experimental data for Al_2O_3–water nanofluids. All data were taken for $d = 36$ nm nanofluids

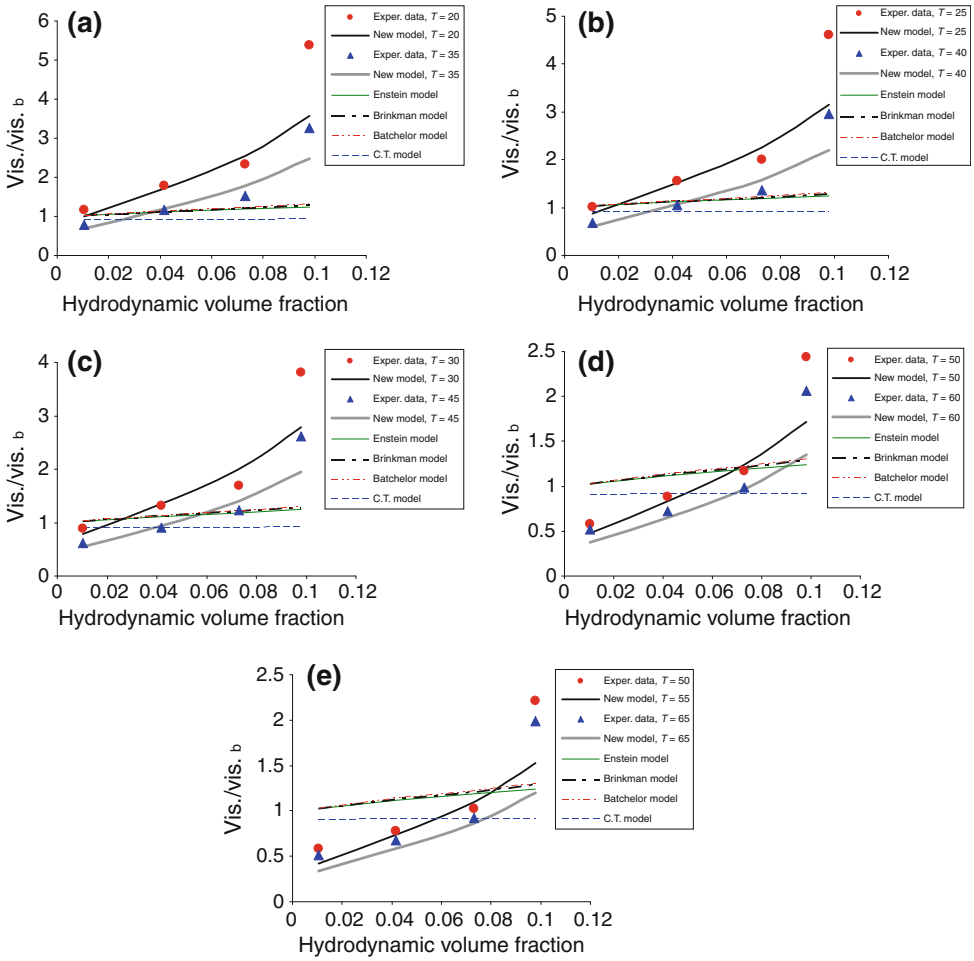

Fig. 4 Model prediction and experimental data for Al_2O_3–water nanofluids. All data were taken for $d = 47$ nm nanofluids

Results and discussion

Figures 1a, b and 2a, b compare the predictions made by the model described in Eq. 12 with experimental data for Al_2O_3–water nanofluid systems. It is clear from the figures that the model is in good agreement with the experimental data. It will now be shown that the model predicts the viscosity of the nanofluid as temperature is changed.

Figures 3a–e and 4a–e show how the viscosity changes with nanoparticle volume fraction for a range of temperatures. These figures show that the model works well as temperature is varied. The model is nonlinear with nanoparticle loading (i.e., nanoparticle volume fraction), while other models are linear. The Einstein, Brinkman, Batchelor, and Nguyen models only consider the effect of particle concentration, but it is clear that there are other factors that must be considered. The model described in Eq. 12 also takes particle diameter, capping layer thickness, and nanofluid temperature into consideration. Therefore, its prediction is much more accurate.

Conclusions

A new model for the viscosity of nanofluids based on dimensionless groups has been presented. Compared to other models that describe the viscosity of nanofluids, the new expression considers not only the volume fraction and base fluid viscosity, but it also takes the nanoparticle size, effect of the capping layer, and temperature of the nanofluid into consideration. This new model is simple in form, yet it is in good agreement with experimental data for Al_2O_3–water nanofluids. The model generates the non-linear behavior that is seen in the viscosity of nanofluids and accurately predicts viscosity due to temperature changes.

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